Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind,
Pune – 16
End Semester Examination: March/April 2025
Faculty: Science and Technology

Program: BSc Gen03 **Semester: VI**
Program (Specific):B.Sc. **Course Type: Core**
Class: T.Y.B.Sc (Mathematics) **Max. Marks: 35**
Name of the Course: Ring Theory **Course Code: 24-MT-363**
Paper no.: III **Time: 2Hrs**

Instructions to the candidate:

1) ***There are 3 sections in the question paper. Write each section on separate page.***
2) ***All sections are compulsory.***
3) ***Figures to the right indicate full marks.***
4) ***Draw a well labelled diagram wherever necessary.***

SECTION: A

Q1) Solve any five of the following. **(10 Marks)**

a) Find all zeros of $x^3 + 2x + 2$ in $\mathbb{Z}_7$.
b) Show that the map $\phi : \mathbb{Z} \to \mathbb{Z}$ defined by $\phi(x) = 3x; \forall x \in \mathbb{Z}$ is not a ring homomorphism.
c) Give an example of finite commutative ring.
d) In $\mathbb{Z}[i]$, show that 5 is not irreducible.
e) Define a unit of a ring. Describe all units of $\mathbb{Z}$.
f) Find the characteristic of the following rings:
i) $\mathbb{Z}_5$ ii) $3\mathbb{Z}$
g) Is $\{0\}$ a prime ideal of $\mathbb{Z}_4$? Justify.

SECTION: B

Q.2) Solve any three of the following. **(Marks 15)**

a) Find the quotient $q(x)$ and remainder $r(x)$ when $f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$ is divided by $g(x) = x^2 - 2x + 3$ in $\mathbb{Z}_5[x]$.

b) If p is an irreducible element in a Principal Ideal Domain D and $p|ab$ for a, b $\in$ D then prove that either $p\,|a$ or $p|b$.

c) Show that f(x) = $25x^5 - 9x^4 - 3x^2 - 12$ is an irreducible over $\mathbb{Q}$.

d) Find all prime and maximal ideals in $\mathbb{Z}_6$.

e) Let $f(x) \in F[x]$ be a polynomial of degree 2 or 3 then prove that $f(x)$ is reducible over F if and only if $f(x)$ has zero in F, where F is a field.

SECTION: C

Q.3) Solve any one of the following. (Marks 10)

a) Prove that every PID is a UFD.

b) i) Find σ and ρ in $\mathbb{Z}[i]$ such that $\alpha = \beta\sigma + \rho$ with $N(\rho) < N(\beta)$, where $\alpha = 7+2i$ and $\beta = 3-4i$ in $\mathbb{Z}[i]$.

ii) If R is a ring with unity 1 and $\emptyset : R \to R'$ is a ring homomorphism then prove that $\emptyset(1)$ is an unity in $\emptyset[R]$. If S is a subring of R then prove that $\emptyset[S]$ is a subring of R'.

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